Homework 5

March 4, 2017

1. This problem involves modeling and inference for count data recorded over discrete time, using a Bayesian formulation for a Markov dependent mixture of K Poisson distributions. Specifically, denoting the data by $\boldsymbol{y} = \{y_1, ..., y_T\}$, the Bayesian model is given by:

$$
\begin{aligned}
& y_t | z_t, \lambda_1, ..., \lambda_K \sim Poisson(y_t | \lambda_{z_t}), t = 1, ..., T \\
& \boldsymbol{z} = (z_2, ..., z_T) | \boldsymbol{Q} \sim \prod_{t=1}^{T-1} P(z_{t+1} | z_t, \boldsymbol{Q}) = \prod_{t=1}^{T-1} q_{z_t, z_{t+1}} \\
& \lambda_j \overset{ind}{\sim} Gamma(c_j, d_j), j = 1, ..., K \\
& \boldsymbol{q}_i \overset{ind}{\sim} Dirichlet(a_{i1}, ..., a_{iK}).
\end{aligned}
$$

   $\boldsymbol{q}_i$ is the ith row of the transition matrix $\boldsymbol{Q}$. To avoid issues with identifiability, fix the first hidden state, for instance, set $P(z_1 = 1) = 1$. For an application of the model consider the fetal lamb movement data available in Table 1 from Leroux and Puterman (1992). To specify the parameters of the priors for the $\boldsymbol{\lambda}_j$ and the $\boldsymbol{q}_i$, you can follow the guidelines in Chib (1996), which presents a Bayesian analysis of this data set using the model above.

   Consider the case of the two-state Poisson HMM ($K = 2$). Develop and implement a MCMC algorithm to sample from the posterior distribution of the model. Obtain posterior estimates of all the parameters.

1. Chib, S. (1996). "Calculating posterior distributions and modal estimates in Markov mixture models." *Journal of Econometrics*, **75**, 79-97.
2. Leroux, B.G. and Puterman, M.L. (1992). "Maximum-Penalized-Likelihood Estimation for Independent and Markov-Dependent Mixture Models." *Biometrics*, **48**, 545-558.