Homework 2

January 19, 2017

1. **Use:** Let $\boldsymbol{X} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ and $\boldsymbol{L}$ be the Cholesky factor of $\boldsymbol{\Sigma}$ that is a unique lower triangular $n \times n$ matrix such that $\boldsymbol{\Sigma} = \boldsymbol{L}\boldsymbol{L}'$. Let $\boldsymbol{z} = (z_1, ..., z_n)'$ be such that z_i's are i.i.d $N(0,1)$, then $\boldsymbol{X} = \boldsymbol{\mu} + \boldsymbol{L}\boldsymbol{z}$. This representation can be used to draw random sample from $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$.

   Use the above fact to simulate sample paths from the Gaussian process $W = \{W_t : t \in \mathcal{R}\}$ with mean 0 and the covariance function given by

   $$Cov(W_t, W_s) = \sigma^2 \exp(-\phi|s-t|^{\alpha}), \phi > 0, \sigma^2 > 0, 0 < \alpha < 2.$$

   Use different combinations of σ^2, ϕ, α to simulate Gaussian sample paths. Discuss how sample path properties are dependent on the values of these parameters.

2. Consider the Gaussian process regression setting with a single (continuous) covariate. Simulate $x_i \sim N(0,1)$ and simulate y_i from the following equation

   $$y_i = g(x_i) + \epsilon_i, \;\; \epsilon_i \sim N(0, \sigma^2), i = 1, ..., n,$$

   with $g(x) = 0.3 + 0.4x + 0.5\sin(2.7x) + \frac{1.1}{1+x^2}$. Take $n = 1000$.
   (1) Fit a Gaussian process model with the exponential correlation function.
   (2) Fit a sparse Gaussian process model with the exponential correlation function with $m = 100$ knots.
   (3) Fit a kernel convolution model with your favorite kernel and with $m = 100$ knots.

   In each of the cases provide the full inference on the model parameters and report the run time for 5000 MCMC iterations.